

5ths Machine: Equal Temperament and the Circle of Fifths

Circle of Fifths/Fourths Diagrams originated in Russia in the 1670s and gradually took their modern shape along with the transition from meantone to equal temperament (ET) for keyboard tuning. These diagrams are only meaningful in the ET system, where the standard octave contains twelve equally spaced half steps tuned in slightly imperfect fifths. Those twelve notes naturally map to a clock face. The American standard portrays the sharps (fifths) stepping clockwise, the flats (fourths) counter-clockwise. At the 6:00 position there is a doubling-up where both the F sharp (six sharps) and the G flat (six flats) key signatures are shown. Many diagrams carry this double display up one more place on each side, so that B (five sharps) and C flat (seven flats) are shown at the 5:00 position, and D flat (five flats) and C sharp (seven sharps) at the 7:00 position.

As shown in our LAMP™ apps PentaMonochord™ and MeanTones™, both Pythagorean and meantone tunings are based on a hypothetical infinite line of ascending perfect fifths, with sharps extending to the right and flats extending to the left. In the first case, the fifths are slightly too large to form a closed circle from twelve positions, and in the second case they are slightly too small. Therefore, for both these systems, a sharp/flat interchange is not possible. Near-enharmonic tones such as F♯ and G♭ are not the same pitch.

By contrast, ET is deliberately set up with fifths that are right-sized to form a circle and allow for enharmonic change. The tradeoff for this flexibility is that no intervals in ET besides the octaves themselves are properly tuned according to the overtone series. The 'perfect' fifths in ET are somewhat smaller than their proper ratio of 3:2, while the major thirds are quite a bit larger than their proper ratio of 5:4. These discrepancies are definitely audible, and may sound discordant to listeners when compared with justly-tuned passages from, say, a string quartet or an *a capella* singing group.

At start-up, the 5ths Machine screen resembles an ordinary circle of fifths/fourths diagram. The special feature of this representation (besides the gears) is that it continues clockwise to add sharps for five full circuits when following the circle of fifths, or counter-clockwise to add flats for five full circuits when following the circle of fourths. Instead of incrementing key signatures to a maximum of six **chromatic signs** (sharps or flats), it continues all the way to 60, in a mind-expanding exercise *ad absurdum*.

There is no practical value to this machine for extending and complicating musical key signatures, but it is a useful tool for deeper understanding of the Western diatonic system. The circle spinning around and around shows how ET limits us to twelve pitch classes represented by the clock positions. Due to octave equivalence, those twelve notes could be placed within a one-hand span on the piano keyboard, even if the hypothetical key signatures contained a million or a trillion signs.

Take a tour of the 5ths Machine with Manual Step, Auto Step, or Auto Fast. The symbol in the center toggles between sharps and flats, and thus between the clockwise circle of fifths and the counter-clockwise circle of fourths. Press “Quiz” to baffle your brain with key signatures you’ve never seen before! ♦