MeanTones: Quarter-comma Meantone Temperament Explained

Meantone Temperament (MT), in some form, was the standard tuning system for musical keyboards for at least 250 years (late 1400s through early 1700s) in Britain and Europe. The designation “meantone” refers to the stipulation that a major third is divided into two equal parts, because all the whole steps are the same size. Technically speaking, equal temperament (ET) is a special case of MT where the half steps are equal as well, but in practice “meantone” is used to refer to older systems with unequal half steps. **Meantone Quarter-comma D-symmetric** was a simple version which was first fully described (though not named) by the organist Michael Praetorius in 1619. MeanTones™ demonstrates this scale using the layout of the modern piano, which was also commonly used for keyboards of the period.

The first step in constructing a meantone scale on a standard keyboard is to choose pitch names for the twelve notes in the octave. Start with a hypothetical line of ascending perfect fifths named according to the seven-letter musical alphabet:

...G♭ D♭ A♭ E♭ B♭ F C G **D** A E B F♯ C♯ G♯ D♯ A♯...

It is crucial to realize that all of the above are distinct pitches, even when adjusted to fall in the same octave. Each of the sharped notes on the right (e.g. C♯) sounds a bit higher than its counterpart (in this case D♭) on the left. The pattern of the line is clearly D-symmetric (as is the piano keyboard), but in order to fit the set onto one octave of the keyboard, twelve pitches in sequence including the seven natural ‘white keys’ and five of the altered ‘black keys’ must be selected from the line. The most common solution is to retain E♭, B♭, F♯, C♯, and G♯ as the altered notes, skewing the symmetry to the right and eliminating all flats from A♭ on to the left, and all sharps from D♯ on to the right.

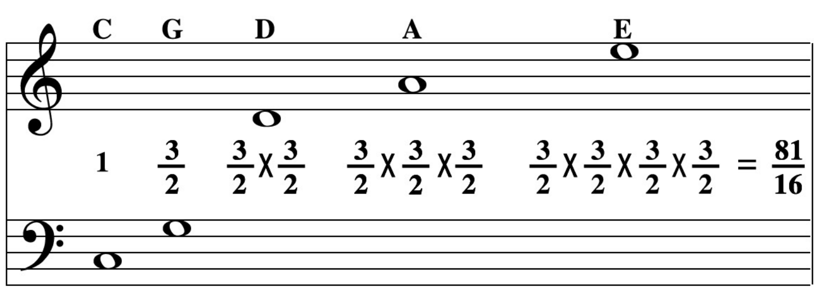
...G♭ D♭ A♭ ¦E♭ B♭ F C G **D** A E B F♯ C♯ G♯¦ D♯ A♯...

The physics of music is based on fractions and ratios. Musical intervals are both defined and perceived as frequency ratios, and the simplest ratios are the most resonant: the perfect intervals octave and fifth are 2:1 and 3:2, respectively. To quickly review, the value of a fraction is the numerator divided by the denominator. The **reciprocal** of a fraction is found by exchanging the numerator and denominator, which is equivalent to dividing the number 1 by the fraction. Each fraction has an equivalent ratio: the statement 3/2 = 3:2 is read “three halves equals three to two” and three halves of three halves is 3/2 x 3/2 = 9/4. Middle-range musical frequencies are actually measured in hundreds of waves per second, but, for calculation purposes, any starting point may be called ‘1’. Then, the octave above that is 2, and the next octave is 2 x 2, or 4. The fifth above the ‘1’ is 3/2, the fifth above that is 3/2 x 3/2, and so on.

A musical **comma** is a very small interval which represents a discrepancy in pitch between two closely related notes. For example, the **Pythagorean comma** constitutes the difference between C♯ and D♭ in the line above; C♯ is higher because the 3:2 perfect fifths are slightly too large to be formed into a twelve-point circle.

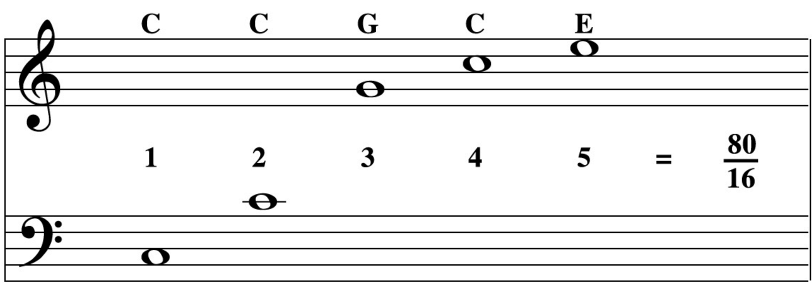
The **syntonic comma** is the source of the problem of sonority that meantone tuning attempts to solve, as explained below.

Start on C and place a sequence of four ascending perfect fifths. Each fifth has the frequency ratio of 3:2 to the one before. Since we are arbitrarily calling the frequency of the starting C ‘1’, the final note E in the stack has the frequency 81/16.



When the C of frequency ‘1’ is produced by plucking a string, a set of higher pitches called the **overtone series** is generated by the vibrations of the string in fractional segments. These overtones are only sometimes heard as distinct pitches, but always affect the **timbre** (quality) of the tone. Any type of instrument produces its own pattern of overtones, always at the same pitches but different in relative volume.

As explained in our PentaMonochord™ app, frequency ratios are the reciprocals of string length ratios. Since the final E in the series is produced by the vibration of 1/5 of the string, it has the frequency 5, which can be written as 80/16.



Here is the dilemma. Even without sophisticated equipment, it is easy to tune a scale using the natural resonance of perfect fifths, and also easy to tune it using the natural resonance of the overtone series. But the first method finds a higher E, and therefore a larger major third, than the second one does. This discrepancy is the syntonic comma, and the problem it represents has been clear to music scholars for over two thousand years.

The **cents system** was introduced by Alexander Ellis in collaboration with Hermann von Helmholtz, to accompany Ellis’s translation of Helmholtz’s *On the Sensations of Tone* into English in 1875. It works like an old-fashioned slide rule, turning logarithmic calculations into simple addition and subtraction. Ellis gave an ET half step the value of 100 cents, so that in ET, all intervals on the piano keyboard are even multiples of 100 according to how many half steps they contain; the major third is 400 cents, the perfect fifth is 700, and the octave is 1200. In this system, the 81/80 syntonic comma equals about 21.5 cents, a small but perceptible difference amounting to approximately an eighth-tone.

To set up a keyboard in meantone quarter-comma temperament, the tuner deliberately makes an adjustment to each fifth to shift it smaller by a quarter of a syntonic comma, or about 5.4 cents in modern terminology. The fifths are now slightly too small to be formed into a twelve-point circle; each of the sharped notes on the right (e.g. C♯) sounds a bit lower than its counterpart on the left (D♭). But in this scheme, the major thirds are in agreement with the overtone series, and most listeners find that tradeoff to be a net gain of harmonious properties.

In general, the harmonies of diatonic music with simpler key signatures are sonorous and smooth in MT. However, there is a downside to a linear system in which the two ends do not connect. Piano students today are taught that each black key is known by two different names. (In actual fact, in ET, any key can be called by literally any number of different names, as shown in our 5ths Machine™ app.) But in MT, each black key represents one pitch name only, and therefore some intervals cannot be **spelled correctly** (named with the proper letters for the interval type). This is not an abstract concern; those intervals will disturb the listener.

Suppose a piece is written in the key of E flat. The composer will use the subdominant triad (the IV chord) at some point, and here is where repercussions begin. The proper root of this chord would be A♭, but that note is not available on our keyboard. Therefore, the chord must be misspelled as G♯ - C - E♭. Those outer notes comprise the famous wolf fifth between G♯ and E♭, which howls painfully because it is not a perfect fifth at all, but a diminished sixth. Its inversion, the wolf fourth E♭ - G♯, is likewise not a perfect fourth, but an augmented third. There are four wolf thirds which are actually diminished fourths. In practice, this meant that the key of E flat was hardly ever used in compositions for keyboard instruments. Eventually musicians became frustrated with such limitations and the musical culture gravitated towards ET.

Remember that a **wolf interval** is not the same thing as a **wolf tone.** A **wolf interval** is a combination of two notes, usually on a keyboard instrument, which sounds unexpectedly dissonant and harsh due to a system of tuning. A **wolf tone**, or **wolf note**, is a ‘howling’ oscillation created on a stringed instrument when the pitch of the note played nearly matches the natural resonating frequency of the instrument’s body.

The MeanTones keyboard gives two synthesizer-generated choices for playing intervals. The piano sound was chosen for its clarity of tone, while the cheesy organ (our tongue-in-cheek name for a saxophone patch) provides a sustained tone without **vibrato** (pitch waver), making the irritating nature of the wolf intervals easier to hear. No attempt has been made in this app to replicate the sound of period instruments. The effect of meantone tuning cannot be fully evaluated or appreciated using any electronic device; the only way to authentically experience these scales and intervals is to hear or play a period instrument in a resonant room. MeanTones hopes mainly to introduce the theory of this fascinating, but daunting, subject in a relatively painless way.

Press a note on the keyboard to hear an interval played in the meantone quarter-comma system, while the chart above the keyboard lets you compare its specifics with the same interval in ET. Switch between fifths, fourths, and thirds to try all the options. Press “Play Game” to spell-check intervals on your way to building a brick house. In order to finish it, you’ll have to keep track of those wolves! ♦