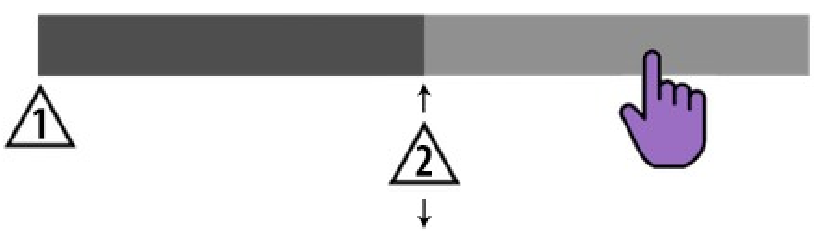
PentaMonochord: Pythagoreans and the Pentatonic Scale

Since ca. 500 BCE, the **Pythagorean Monochord** has been used to study musical intervals. In its simplest form, the monochord is a single string stretched between two fixed ends with sufficient tension so that when the string is plucked it makes a sound with a musical pitch. The monochord also allows for the placement of a movable bridge underneath the string. Where the bridge touches the string, it shortens the **speaking length** (the vibrating portion) of the string to the distance between its contact point and one of the fixed ends. Finally, setting up the monochord requires a measuring system to specify the bridge placements.

The PentaMonochord™ is a virtual representation of a Pythagorean monochord using IntervalBlocks™. Each block functions as though it contained two virtual bridges and a finger to pluck the string twice; this happens automatically when a block is placed on the string. The speaking length for the lower note is the whole block, while the speaking length for the higher note is the lighter-colored portion. As the block position moves to the right, the root pitch gets higher and the block's calipers automatically remeasure it to a shorter length.

A musical interval consists of two distinct pitches. The definition of the interval remains the same whether these pitches are sounded simultaneously or consecutively. All the intervals on the PentaMonochord are heard consecutively, and the virtual finger always plucks the string to the right of the rightmost virtual bridge.



- The first virtual bridge is located at the left edge in the block, or is not needed if the block covers the whole string.
- The second virtual bridge is located at the color change in the block, and can be placed on and pulled off.
- The string is plucked to the right of the second virtual bridge.

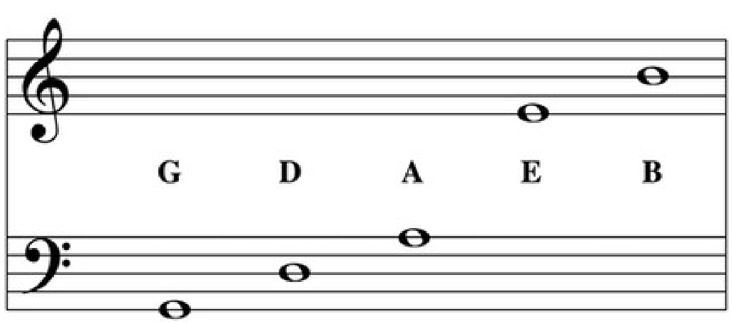
The five-note **Pentatonic Scale** is approximately equivalent to the black keys on the piano. Its structure of pitches is deeply rooted in the physics of music. These relationships are based on fractions and ratios.

To quickly review, the value of a fraction is the numerator divided by the denominator. The **reciprocal** of a fraction is found by exchanging the numerator and denominator, which is equivalent to dividing the number 1 by the fraction. Each fraction has an equivalent ratio — the statement 3/2 = 3:2 is read “three halves equals three to two”. Musical intervals are both defined and perceived as frequency ratios, and the simplest ratios are the most resonant thanks to the sympathetic vibrations of their overtones. The two intervals described below, along with their inversions, are uniquely designated as 'perfect' in music theory. This is due to the effect of this resonance on the mechanism of the inner ear that perceives vibration frequency as musical pitch.

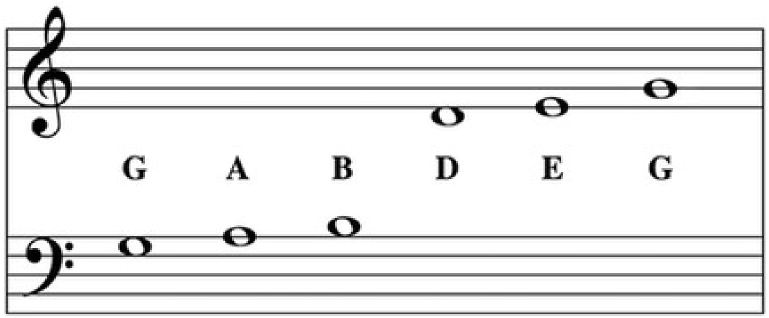
The interval of a tone and the tone twice its frequency (a ratio of 2:1) is called a perfect octave, which is sensed by humans as an equivalence of sounds. A musical unison passage is defined as instruments or voices playing or singing the same line of notes together, whether they are at exactly the same pitch or in a different octave. Adding octaves above or below a certain pitch creates abundant sonority, but there are no new notes to be found, as if a pianist were limited to playing only the eight B’s or the seven E’s on the keyboard.

The interval of a tone and the tone 1½ times its frequency (a ratio of 3:2) is called a perfect fifth, and the human ear perceives this interval as a natural correlation — not an equivalence — of the two pitches. By stacking fifths above or below a certain pitch, it is possible to construct an infinite catalog of notes. Some of the world’s most basic scales derive from stacks of fifths.

To illustrate this, start on G and place a sequence of four ascending 3:2 perfect fifths:



Now, using octave equivalence, order them in the same pitch range. This makes a singable scale out of the perfect fifth building blocks:



Voilà, a G pentatonic scale!

These simple intervals and scales were well known to musicians for thousands of years before Pythagoras, but with the invention of the Pythagorean monochord it was possible to define them precisely. The Pythagoreans achieved extraordinary discoveries using only common materials and tools, and their findings were confirmed by science and technology over two thousand years later. Except that the ratios were backwards! The numerical relationship of the frequencies of two pitches is not the ratio of string lengths on the monochord for those pitches, but the *reciprocal* of that ratio.

The traditional Vietnamese instrument called a *dan bau* is also a monochord. This instrument is fashioned with a flexible rod to bend the pitch of the single string, similar to the 'whammy bar' used on electric guitars. A modern *dan bau* also has a pickup to enable electronic amplification. The *dan bau* is effective for performance, unlike the Pythagorean monochord which is useful only for the study of intervals.

'Therapy monochords', which are actually multi-stringed instruments tuned to a single pitch, or to a very limited number of pitches. Strumming them creates vibrations thought to be healing. It is incorrect to call this type of instrument a Pythagorean monochord.

With a Pythagorean monochord, pluck the entire string, and then, after placing a bridge in the exact middle, pluck it again. Or, do the same thing with our virtual PentaMonochord by placing a blue IntervalBlock at G2 on the left edge. Either way, you will hear an ascending octave produced by this 2:1 ratio of speaking lengths. As we now know, the frequency ratio of the octave is 1:2, with the upper note twice as high. A monochord, along with bridges and calipers, remains a fine tool for determining the frequency ratio of any interval; it is only necessary to reverse the ratios of the speaking lengths of the string.

PentaMonochord IntervalBlocks

Perfect Octave 2:1 Up

Circle a note pointer with the lasso at the left edge to create an ascending octave

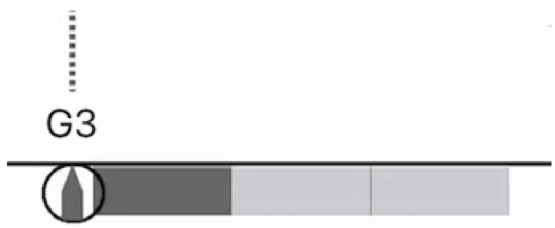
Perfect Fifth 3:2 Up

Circle a note pointer with the lasso at the left edge to create an ascending fifth

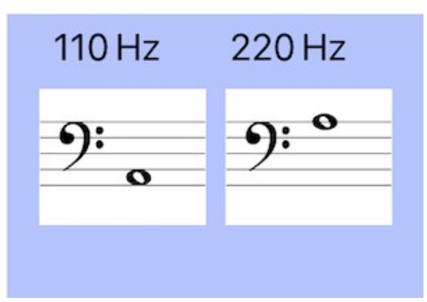
Perfect Octave 2:1 Down

Circle a note pointer with the lasso at the right edge to create a descending octave

As soon as you begin to drag a block from the stack below the string, a lasso appears at its right or left edge. At the same time, a pointer the color of the block appears at each location on the string where the block can be placed. Use the lasso to circle whichever pointer you choose, then drop the block.



Use IntervalBlocks to build a two-octave G pentatonic scale on the PentaMonochord using only octaves and fifths. Touch any block to see and hear its interval again, and watch the Hertz-box to see how the frequencies of the two notes relate inversely to their proportions on the IntervalBlock.



Touch the note labels any time to play a pentatonic tune. Press “Switch to Game” to join the Pythagorean pantheon by building the whole scale with the minimum number of steps. ♦