# 5ths Machine: How To Solve the Quiz Questions

## Introduction and Requirements

The 5ths Machine™ Quiz generates random questions of the form “Which key has [some number of sharps or flats]?” The numbers in the questions range from 7 to 60, and the answers are in the form of [letter name] [n-tuple] [sharps/flats], where the sharps/flats choice matches the question. The quiz assumes:

- Understanding of the mapping of the circle of fifths/fourths to a clock face
- Knowledge of the ordinary key signatures for the twelve major scales
- Knowledge of the standard order of sharps and of flats in key signatures
- Knowledge of the piano keyboard, and access to one or an image of one
- Proficiency in arithmetic and familiarity with algebraic concepts

The first step is to become fluent at resolving an extended key signature with an excessive number of **chromatic signs** (i.e. sharps or flats) to its **simple enharmonic scale**, which is the equivalent scale with an ordinary key signature. For our purposes, 7 chromatic signs qualifies as an excessive number. To simplify a key signature with a number of signs between 7 and 11, subtract the number from 12 and flip to the opposite chromatic sign. To simplify a key signature with 12 or more signs, divide the number by 12 and use the remainder if it is 6 or less. If the remainder is larger than 6, subtract it from 12 and flip the sign.

*For identifying simple enharmonic scales, here are four practice examples*:

Q: What is the simple enharmonic scale for 8 sharps?
A: Subtract 8 from 12 and flip the sign: 4 flats, or A flat.

Q: What is the simple enharmonic scale for 10 flats?
A: Subtract 10 from 12 and flip the sign: 2 sharps, or D.

Q: What is the simple enharmonic scale for 39 sharps?
A: Divide 39 by 12 and use the remainder 3: 3 sharps, or A.

Q: What is the simple enharmonic scale for 47 flats?
A: Divide 47 by 12, subtract the remainder 11 from 12, and flip the sign: 1 sharp, or G.

## The system of sevens

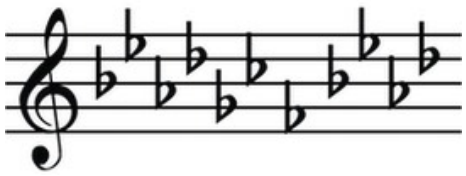
C natural is the blank key signature, C sharp has 7 sharps, and C flat has 7 flats. This demonstrates that when a tonic note is a half step higher or lower than the starting one and the letter name is the same, the number of chromatic signs differs by 7 — sharps if higher, flats if lower — because each of the 7 alphabet letters in the scale was adjusted by that half step. Extending the pattern, C double sharp has 14 sharps, C double flat has 14 flats, etc.

To specify a key signature with 7 or more chromatic signs using the system of sevens, divide the number of signs by 7 to produce an integer quotient and a remainder. The remainder, with the sign in question, yields an ordinary key signature. That key signature letter is the alphabetic portion of the answer. The integer quotient represents the number of half-step shifts, which yields the numeric portion of the answer where 2 is double, 3 is triple, etc. (Note that if the ordinary key signature already contains a chromatic sign, 1 must be added to the numeric portion.)

*Here are two sample quiz questions solved using the system of sevens, checked against the simple enharmonic scale, and double-checked by writing out the extended key signatures:*

**Which key has 11 flats?**
Use system of sevens: *11 ÷ 7 = 1 remainder 4*
1 represents 1 half-step shift in the flats direction and 4 represents the ordinary key signature of 4 flats. The tonic of 4 flats is A♭, which contains a chromatic sign, so the answer after adding 1 additional half-step shift is A double flat.

Check by resolving the simple enharmonic scale for 11 flats to G and tapping a piano keyboard to verify that the note A𝄫 is enharmonic with the note G. Double-check by writing 11 flats in the standard order and counting the accidentals for each letter.



A, B, D and E are all double flatted. C, F, and G are each single flatted.

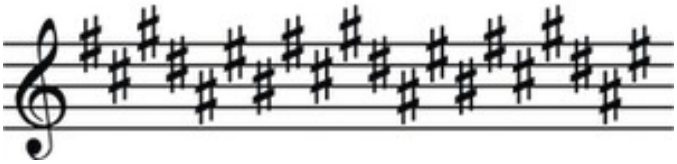
These alterations produce the pitches enharmonic with G, A, B, C, D, E, and F♯, which are the notes in the G scale.

**Which key has 20 sharps?**

Use system of sevens: *20 ÷ 7 = 2 remainder 6*

2 represents 2 half-step shifts in the sharps direction and 6 represents the ordinary key signature of 6 sharps. The tonic of 6 sharps is F♯, which contains a chromatic sign, so the answer after adding 1 additional half-step shift is F triple sharp.

Check by resolving the simple enharmonic scale for 20 sharps to A flat and tapping a piano keyboard to verify that the note F♯x is enharmonic with the note A♭. Double-check by writing 20 sharps in the standard order and counting the accidentals for each letter.



A, C, D, E, F, and G are all triple sharped. B is double sharped.

These alterations produce the pitches enharmonic with A♭, B♭, C, D♭, E♭, F, and G, which are the notes in the A flat scale.

## The system of twelves

It is easy to find the **result clock position** for an excessive number of chromatic signs, since the circuit starts over at each multiple of 12. However, finding a formula to derive the key signature itself is harder, because the 7 letters of the musical alphabet cannot be treated as numbers due to their inconsistent spacing. In the spirit of musicianly practice, 5ths Machine presents a solution using the piano keyboard as a counting device.

The first step in learning the system of twelves is to become fluent at finding the **alphabet start**, which is associated with the result clock position for the question. To find the result clock position, divide the number of signs in the question by 12 and use the remainder. If the remainder is 0, the result clock position is 12:00 and the alphabet start is C. Otherwise, for a sharps key signature, the result clock position equals the remainder, and, for a flats key signature, the result clock position equals 12 minus the remainder. If the remainder is 6 or less, the alphabet start is the same as the simple enharmonic scale at the clock position. If the remainder is greater than 6, the alphabet start differs by one letter — ahead for flats or back for sharps — from the simple enharmonic scale at the result clock position.

*For identifying result clock positions and alphabet starts, here are four practice examples:*

Q: What is the result clock position and the alphabet start for 17 sharps?
A: Divide 17 by 12 to get the remainder 5. Because the key signature is in sharps, the clock position equals the remainder, or 5:00. The tonic of the simple enharmonic scale at this position is B and the alphabet start for sharps is B.

Q: What is the result clock position and the alphabet start for 27 flats?
A: Divide 27 by 12 to get the remainder 3. Because the key signature is in flats, the clock position equals 12 minus the remainder, or 9:00. The tonic of the simple enharmonic scale at this position is E♭ and the alphabet start for flats is E.

Q: What is the result clock position and the alphabet start for 32 sharps?
A: Divide 32 by 12 to get the remainder 8. Because the key signature is in sharps, the clock position equals the remainder, or 8:00. The tonic of the simple enharmonic scale at this position is A♭ and the alphabet start for sharps is G, because the key at 8:00 is G sharp when expressed in sharps.

Q: What is the result clock position and the alphabet start for 47 flats?
A: Divide 47 by 12 to get the remainder 11. Because the key signature is in flats, the clock position equals 12 minus the remainder, or 1:00. The tonic of the simple enharmonic scale at this position is G and the alphabet start for flats is A, because the key at 1:00 is A double flat when expressed in flats.

To specify a key signature with 12 or more chromatic signs using the system of twelves, divide the number of signs by 12 to produce an integer quotient representing the number of extra circuits and a remainder. Derive the alphabet start from the remainder using the procedure above. Starting with this letter, move one place through the musical alphabet for each extra circuit — ahead for sharps or back for flats — to arrive at the **alphabet result**, which is the alphabetic portion of the answer. Starting at the white key with this letter name on a piano keyboard, tap out half steps, moving up for sharps or down for flats, to the tonic of the simple enharmonic scale to arrive at the numeric portion of the answer.

*Here are three sample quiz questions solved using the system of twelves and checked using the system of sevens:*

**Which key has 24 flats?**
Start with system of twelves: *24 ÷ 12 = 2 remainder 0*
The tonic of the simple enharmonic scale for the blank key signature is C, the result clock position is 12:00, and the alphabet start is C. Adjust C ahead 2 places to find the alphabet result E. Starting on the note E, tap downwards on a piano keyboard and count the half steps to C. There are 4, so the answer is E quadruple flat.

Check with system of sevens: *24 ÷ 7 = 3 remainder 3*
The first 3 represents three half-step shifts in the flats direction and the second 3 represents the ordinary key signature of 3 flats. The tonic of 3 flats is E♭, which contains a chromatic sign, so the answer after adding 1 additional half-step shift is E quadruple flat.

**Which key has 41 sharps?**
Start with system of twelves: *41 ÷ 12 = 3 remainder 5*
The tonic of the simple enharmonic scale for 5 sharps is B, the result clock position is 5:00, and the alphabet start is B. Adjust B back 3 places to find the alphabet result F. Starting on the note F, tap upwards on a piano keyboard and count the half steps to B. There are 6, so the answer is F sextuple sharp.

Check with system of sevens: *41 ÷ 7 = 5 remainder 6*
5 represents 5 half-step shifts in the sharps direction and 6 represents the ordinary key signature of 6 sharps. The tonic of 6 sharps is F♯, which contains a chromatic sign, so the answer after adding 1 additional half-step shift is F sextuple sharp.

**Which key has 58 flats?**
Start with system of twelves: *58 ÷ 12 = 4 remainder 10*
The tonic of the simple enharmonic scale for 10 flats is D, the result clock position is 2:00, and the alphabet start is E. Adjust E ahead 4 places to find the alphabet result B. Starting on the note B, tap downwards on a piano keyboard and count the half steps to D. There are 9, so the answer is B nontuple flat.

Check with system of sevens: *58 ÷ 7 = 8 remainder 2*
8 represents 8 half-step shifts in the flats direction and 2 represents the ordinary key signature of 2 flats. The tonic of 2 flats is B♭, which contains a chromatic sign, so the answer after adding 1 additional half-step shift is B nontuple flat. ◆